

chapter #1

[1]

Introduction

→ what is the signal?

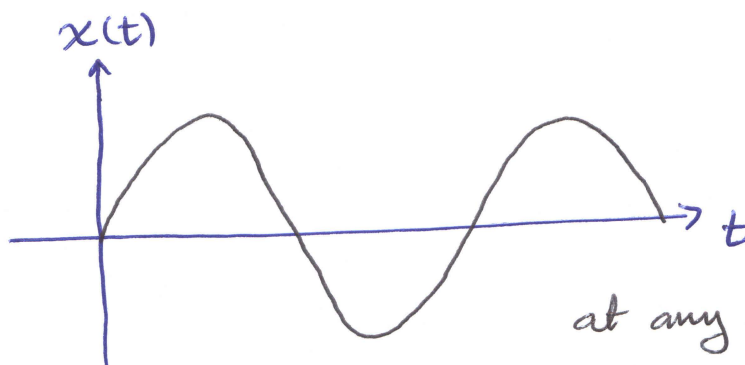
- Signal: are detectable physical quantities or variables by means which message or information can be transmitted.
- signals can be represented mathematically as a function of one or more independent variables.
like time

For example

L) Electrical signals like voltages and currents in the circuit.

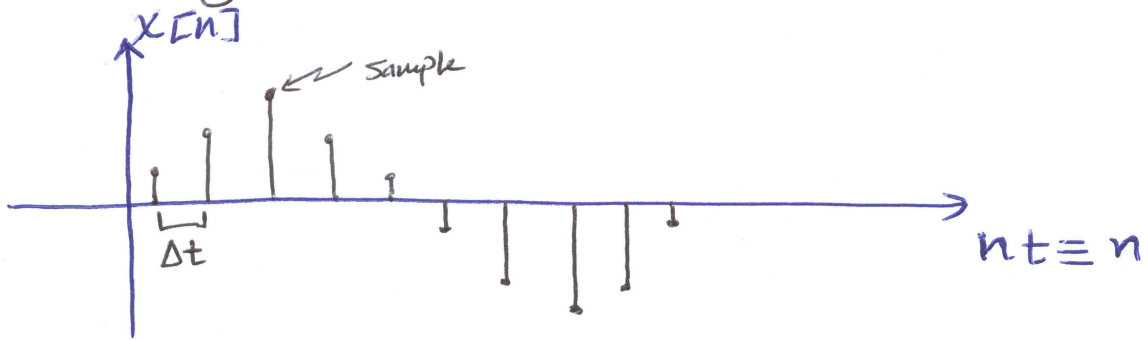
→ we can know the type of signals by determine the type of independent variables. (t)

1) if the independent variables is continuous so the corresponding signal is continuous time (CT)

Example

at any value of t
there is value for $x(t)$

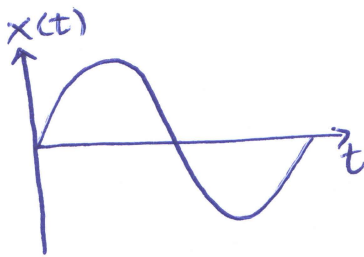
2) if the independent variables is discrete, the corresponding signals is Discrete time (DT)



NOTES:

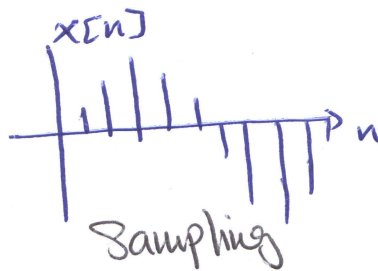
Analoge

Continuous time
Continuous value



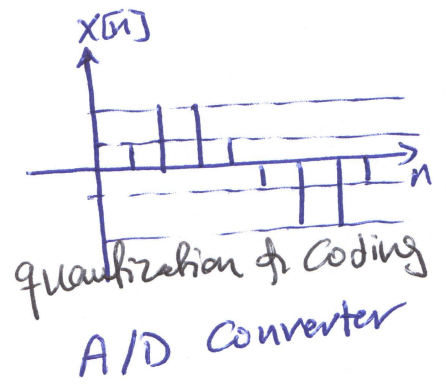
Discrete

Discrete time
Continuous value



Digital

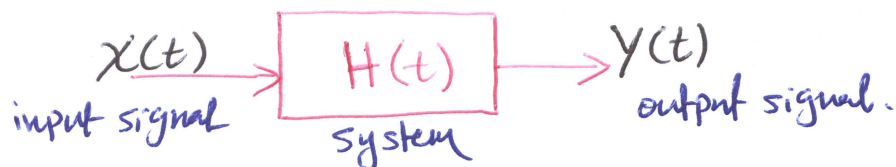
Discrete time
Discrete value



This operation or processing called

→ what is the system?

- Signals may be processed further by systems which may modify them or extract additional information from them.
- System is a black box that transforms input signals to output signals.



1.1 Size of signal 2

(3)

- we can use the energy of the signals as a measure for the size of the signal

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

↪ ABS.

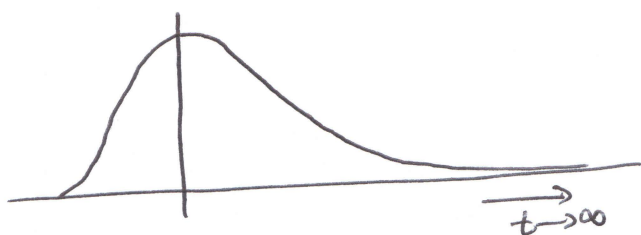
- the signal must be finite to find its energy, which means the signal amplitude $\rightarrow 0$ at $|t| \rightarrow \infty$ (Converge function or system)
- if the signal is not converge, we find the size using the scale of the power of signal.

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

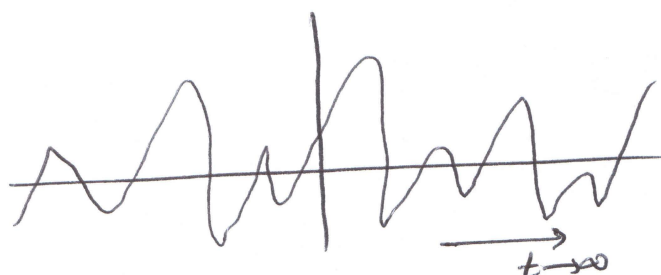
$$P_x = \lim_{T \rightarrow \infty} \frac{E_x}{2T}$$

- P_x is the time average of the energy
- in general, P_x exists iff the signal is periodic

Periodic 2 that means the signal is repeat it self every specific period of time such as sin and cos

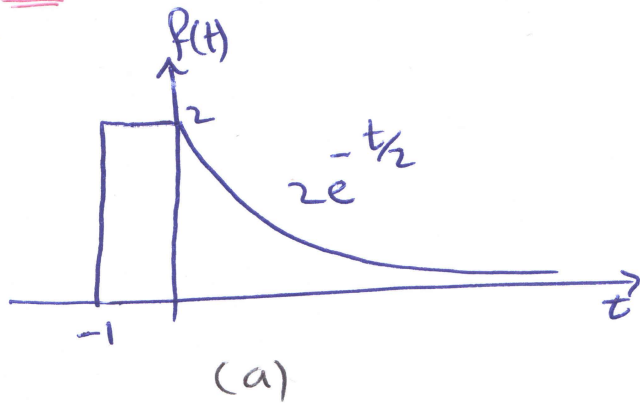
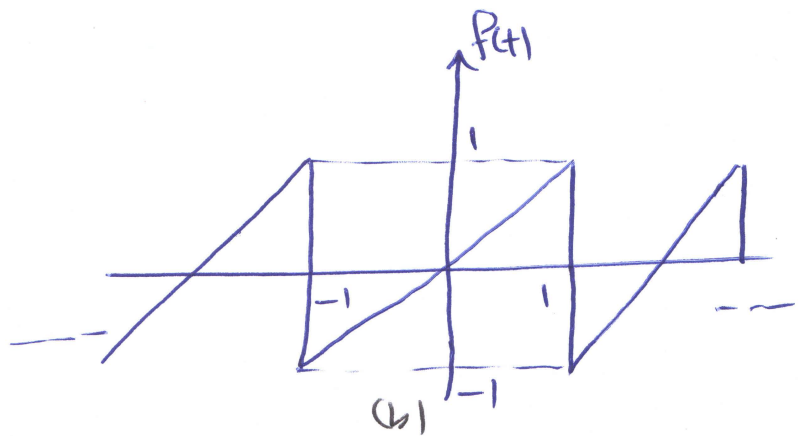


Signal Energy



Signal power

Ex: Determine the suitable measures of these signals [4]



- a) This signal amplitude $\rightarrow 0$ at $t \rightarrow \infty$
that mean this signal Energy so the measure is the energy

$$\begin{aligned} E_p &= \int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-1}^0 (2)^2 dt + \int_0^{\infty} 4e^{-t} dt \\ &= 4t \Big|_{-1}^0 + \frac{4e^{-t}}{-1} \Big|_0^{\infty} \\ &= 0 - (-4) + 0 - (-4) \\ &= \boxed{8} \end{aligned}$$

- b) This signal is periodic signal so the measure is Power

$$\begin{aligned} P_f &= \frac{1}{T} \int_{T/2}^{T/2} f(t)^2 dt \\ &= \frac{1}{2} \int_{-1/2}^{1/2} t^2 dt = \frac{1}{2} \left[\frac{t^3}{3} \right]_{-1/2}^{1/2} \\ &= \frac{1}{2} * \frac{2}{3} = \boxed{\frac{1}{3}} \end{aligned}$$

period of signal is T
[T=2]

$$\begin{aligned} y &= mx \\ m &= \frac{b_1 - b_0}{a_1 - a_0} \\ m &= \frac{1 - (-1)}{1 - (-1)} = \frac{2}{2} \\ &= 1 \\ f(t) &= 1 \cdot t \end{aligned}$$

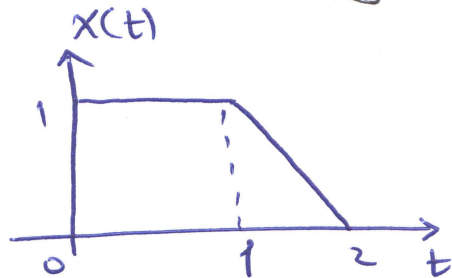
- All periodic signals are power signals but not vice versa.

1.2 Operation on signals :- (Transformation)

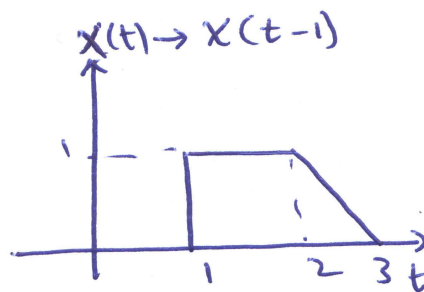
There are 3 possible time transformation

- 1) Time shifting
- 2) Time scaling
- 3) Time reversal (reflection)

(1) Time shifting

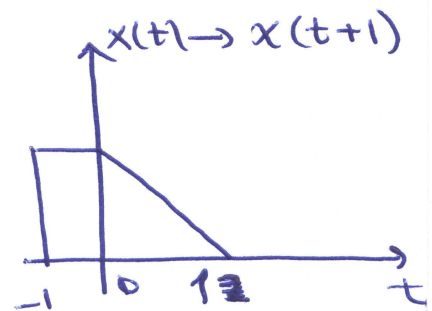


original signal



shift to right

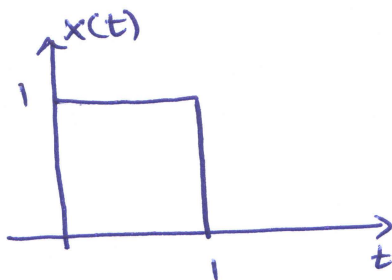
That means make delay or go to the past



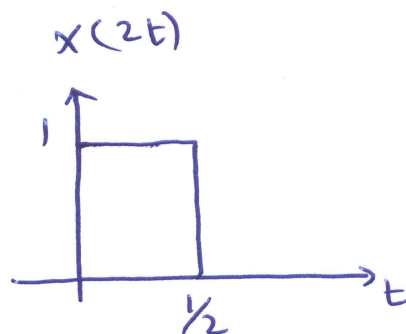
shift to left

means make Advance or go to the future & that impossible in our life

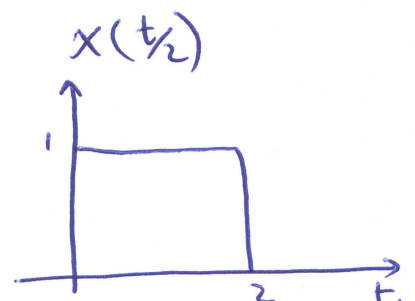
(2) Time scaling



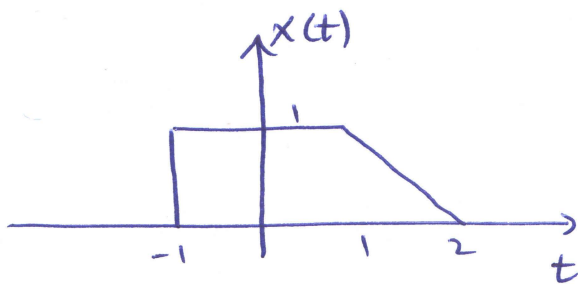
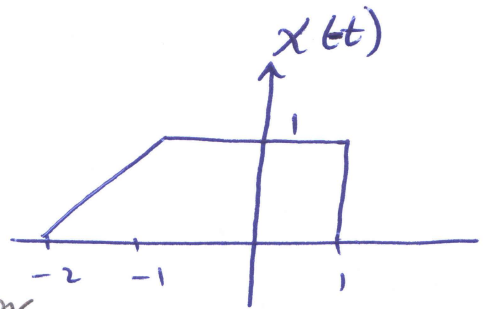
original signal



scaled signal



scaled signal

 $\Rightarrow$ 

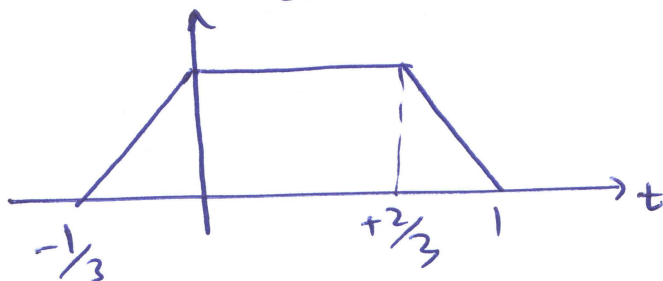
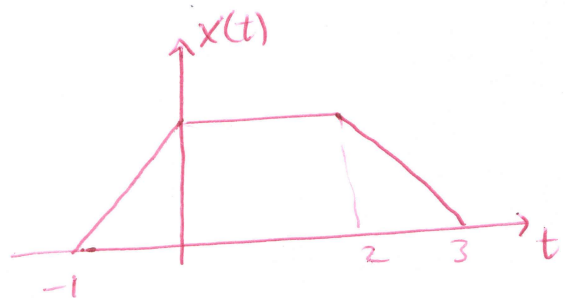
Put mirror
on the y-axis

Exampler

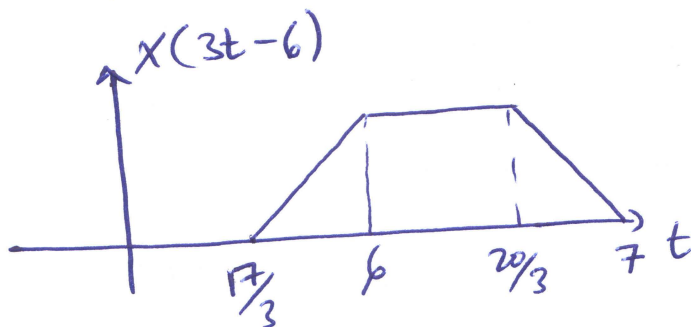
if you have the original signal $x(t)$ as in figure (a), so find the $x(3t-6)$

ملاحظة: الدولتين في هذه الحالة 'تساوي' ① الضرب ② ~~القسمة~~ ~~القسمة~~ ③ ~~القسمة~~ ~~القسمة~~

1) $x(3t)$ $x(\frac{t}{3})$



2) $x(3t-6) \rightarrow$ shift to right.



End



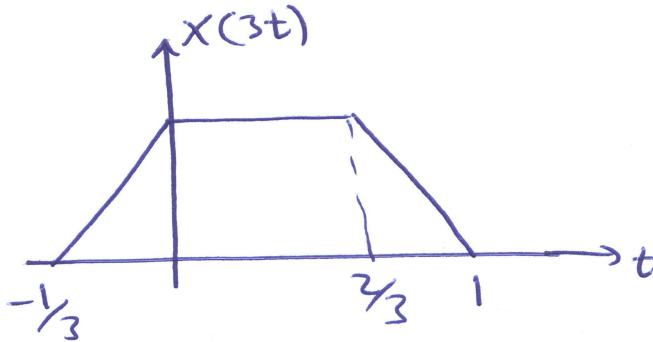
That is wrong solution why?!

Let we see ---

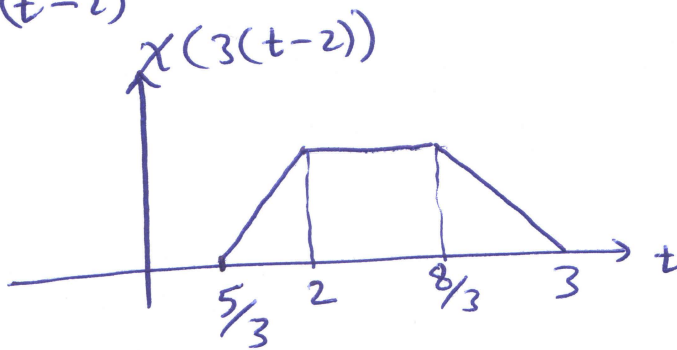
$$x(3t-6) \Rightarrow x(3(t-2))$$

That is correct expression

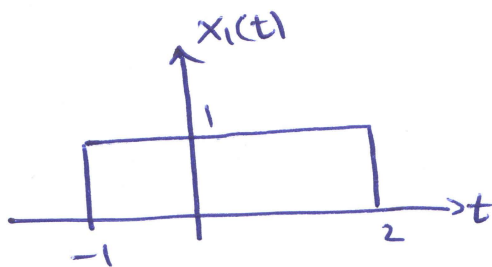
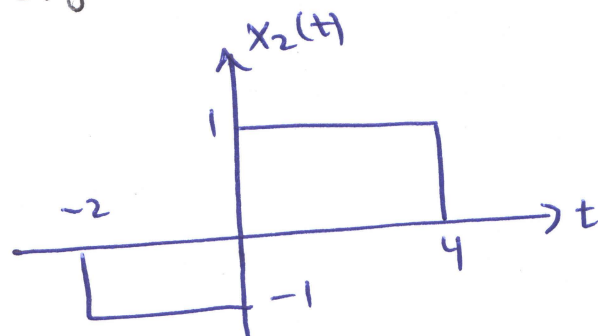
① First make scaling



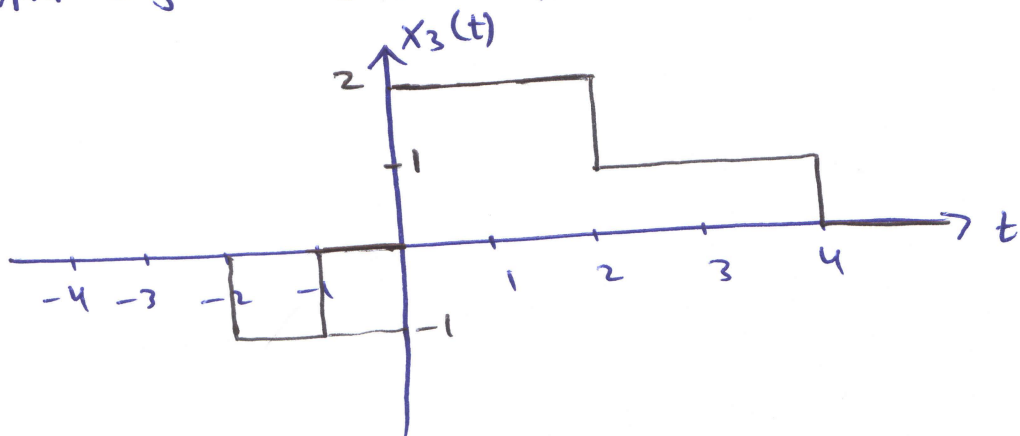
② Second make shift to right by 2 $\Rightarrow$ replace ~~all~~ t by $(t-2)$



⊛ Multiplication of addition signals

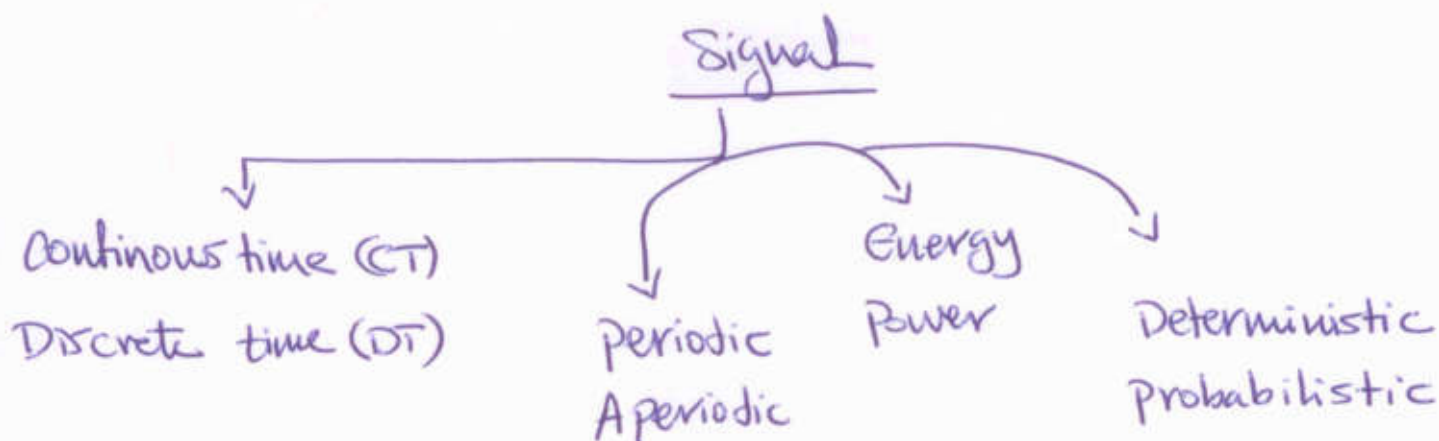


Find the output signal $x_3(t) = x_1(t) + x_2(t)$



1.3 Classification of signals

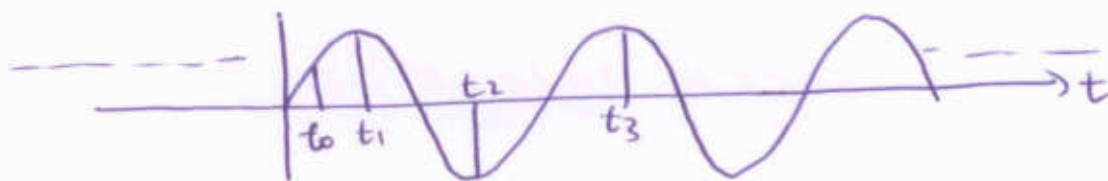
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Let us see what does it mean?!

I) CT and DT signals

- Continuous time signals are functions of a real argument like time $\Rightarrow x(t) = \sin(t)$



at any value of t_n it has a value

- Discrete time signals are functions of an argument that takes values from a discrete set

$$x[n] = \sin(n)$$

$\hookrightarrow$ Integer value



$$\Delta t = 2t_0 - t_1 = T$$

where n is specific value like $n = [\dots, -2, -1, 0, 1, 2, \dots]$

2) Periodic and Aperiodic signals-

9

- The condition of periodicity is

$$x(t) = x(t + nT)$$

$$n = 0, 1, 2, \dots$$

where T is the period of $x(t)$

Notes In continuous signal $\sin$ & $\cos$ are always periodic
but in discrete time $\sin$ & $\cos$ are not.

Example 2

$$x(t) = \underbrace{\sin 2t}_{\text{periodic}} + \underbrace{\sin \pi t}_{\text{periodic}}$$

$$\Rightarrow \left| \frac{\sin \omega_0 t}{P = \frac{2\pi}{\omega_0}} \right|$$

$\therefore$ if $\frac{P_1}{P_2}$ is rational then it is periodic & if it is irrational then it is aperiodic.

$$\left| \frac{P_1}{P_2} = \frac{\text{Int}}{\text{Int}} \right| \text{ (rational number)}$$

$$P_1 = \frac{2\pi}{2} = \pi, \quad P_2 = \frac{2\pi}{\pi} = 2$$

$$\therefore \frac{P_1}{P_2} = \frac{\pi}{2} \text{ is not rational}$$

$\therefore x(t)$ is aperiodic signal.

Example ②:

(10)

$$X(t) = \sin \frac{\pi}{3} t + 2 \cos \frac{8\pi}{3} t$$

في المثال، يطلب منا أن نجد إذا كانت $X(t)$ دورية أم لا.
 periodicity 1. نحتاج أن نجد إذا كانت $X(t)$ دورية أم لا.

$$P_1 = \frac{2\pi}{\frac{\pi}{3}} = 6, \quad P_2 = \frac{2\pi}{\frac{8\pi}{3}} = \frac{6}{8} = \frac{3}{4}$$

$$\therefore \frac{P_1}{P_2} = \frac{6}{\frac{3}{4}} = \frac{8}{1} = \frac{\text{Int}}{\text{Int}} \text{ rational}$$

3. $X(t)$ is periodic, so the period of $X(t)$ equal.

$$P = 8P_2 = P_1 = 6$$

↑ must be equal to be correct answer

- So the last signal is continuous signal like

$$X(t) = \cos \omega_0 t \quad \text{so its period equal } \boxed{P = \frac{2\pi}{\omega_0}}$$

- if the signal is Discrete signal how you can solve it

$$X[n] = \cos \omega_0 nT \Rightarrow \text{its period equal}$$

$$\boxed{\frac{N}{K} = \frac{2\pi}{\omega_0 T}}$$

must be rational

$\omega_0 T \Rightarrow 2\pi$
↑ continuous ↑ discrete

$$\boxed{\frac{N}{K} = \frac{2\pi}{2\pi}}$$

Ex2 $y[n] = \sin[0.3\pi n]$

Discrete Signal

$$\frac{N}{K} = \frac{2\pi}{0.3\pi} = \frac{20}{3} = \frac{\text{int}}{\text{int}} \quad \text{rational}$$

∴ its periodic signal & its period $N=20$

Ex2 $y[n] = \cos\left[\frac{\pi}{4}n\right] + \sin\left[\frac{\pi}{7}n + k\right]$

① ②

$$\frac{N_1}{K} = \frac{2\pi}{\pi/4} = \frac{8}{1} \quad \text{rational} \Rightarrow \text{periodic } N_1 = 8$$

$$\frac{N_2}{K} = \frac{2\pi}{\pi/7} = \frac{14}{1} \quad \text{rational} \Rightarrow \text{periodic } N_2 = 14$$

$$\frac{N_1}{N_2} = \frac{8}{14} \quad \text{rational}$$

So $x[n]$ is periodic & its period equal

$$N = \frac{4}{8}N_2 = \frac{7}{14}N_1 = 56$$

VIP2

1) $\sin \omega_0 t = \cos(\omega_0 t - \frac{\pi}{2})$

2) $\cos \omega_0 t = \sin(\omega_0 t + \frac{\pi}{2})$

3) $e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t$

↳ it is always periodic & its period equal $P = \frac{2\pi}{\omega_0}$

Ex 2

$$x(t) = \sin \frac{2\pi}{5} t \cdot \cos \frac{4\pi}{3} t$$

$$\sin x \cdot \cos y = \frac{1}{2} \{ \sin(x+y) + \sin(x-y) \}$$

$$x(t) = \frac{1}{2} \left\{ \sin \frac{26\pi}{15} t + \overset{\text{odd function.}}{\sin \left(-\frac{14\pi}{15} t \right)} \right\}$$

$$P_1 = \frac{2\pi}{\frac{26\pi}{15}} = \frac{30}{26} = \frac{15}{13}$$

$$P_2 = \frac{2\pi}{\frac{14\pi}{15}} = \frac{30}{14} = \frac{15}{7}$$

$$\frac{P_1}{P_2} = \frac{15/13}{15/7} = \frac{7}{13} \quad \text{is rational}$$

$$P = 7P_2 = 13P_1 = \boxed{15} \quad \#$$

Notes: The exponential function is not periodic

Ex:

$$x(t) = e^{j\frac{\pi}{6}t} + e^{j\frac{4\pi}{5}t}$$

$$P_1 = \frac{2\pi}{\frac{\pi}{6}} = 12$$

$$P_2 = \frac{2\pi}{\frac{4\pi}{5}} = \frac{5}{2}$$

$$\frac{P_1}{P_2} = \frac{12}{5/2} = \frac{24}{5}$$

$$P = 5P_1 = 24P_2$$

$$P = 60$$

$\#$

[3] Energy and Power signals:-

[13]

- if the signal has finite energy so it called energy signal
- if the signal has finite power and nonzero so it called power signal.

The power signal have infinite energy $E = \infty$

[4] Deterministic and Probabilistic signals:-

- if the signal has an equation or graphical that describe it and ~~we~~ i can expect the next values for it its called Deterministic signal.
- if the signal has random value and I can't expect its values so its called probabilistic signal.

1.4 Some Useful signal Models

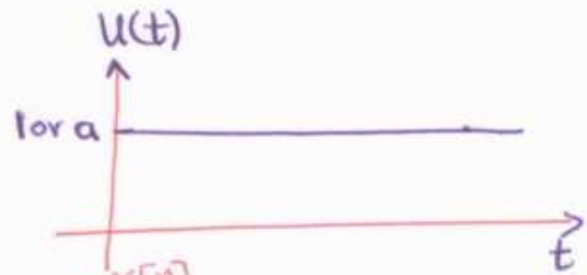
- Step signal
- ramp signal
- Impulse signal
- Exponential signal

Step Function $u(t)$, $u[n]$

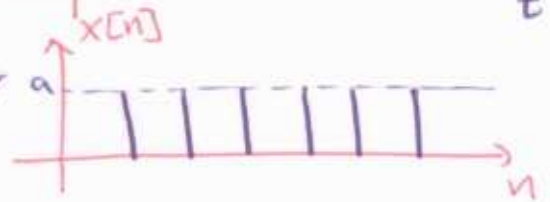
unit step

$$u(t) = \begin{cases} 1 \text{ or } a & t \geq 0 \\ 0 & t < 0 \end{cases}$$

step function



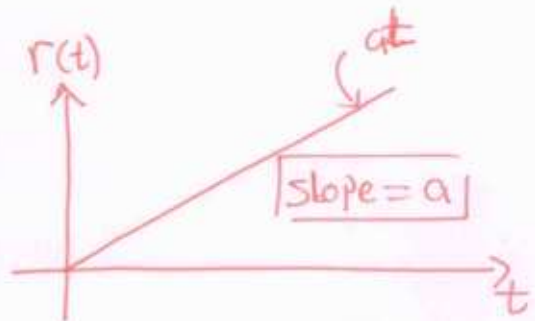
$$u[n] = \begin{cases} 1 \text{ or } a & n \geq 0 \\ 0 & n < 0 \end{cases}$$



Ramp Function $r(t)$

m = slope

$$r(t) = \begin{cases} at & t \geq 0 \\ 0 & t < 0 \end{cases}$$



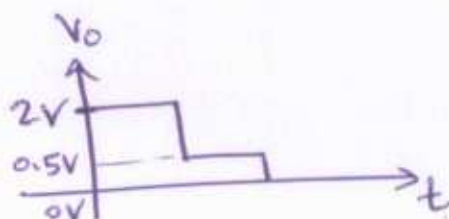
there is relation ship between $r(t)$ & $u(t)$

$$\boxed{\begin{aligned} r(t) &= \int_0^t u(t) dt \\ u(t) &= \frac{dr(t)}{dt} \end{aligned}}$$

در این بخش

Amplitude در این بخش

step در این



[3] Impulse function $\delta(t)$

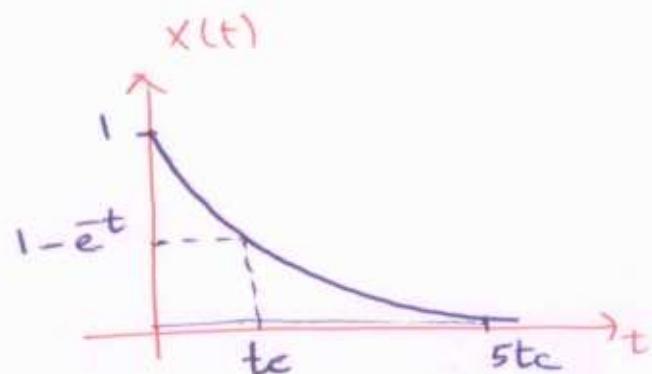
$$\delta(t) = \begin{cases} \infty & t=0 \\ 0 & t \neq 0 \end{cases}$$



[4] Real and Complex exponential

- real exponential function -

$$x(t) = e^{-at}$$



$$t_c = \frac{1}{a}$$

$\hookrightarrow$ is called time constant which is the time required to decrease the amplitude $x(t)$ by (e^{-1})

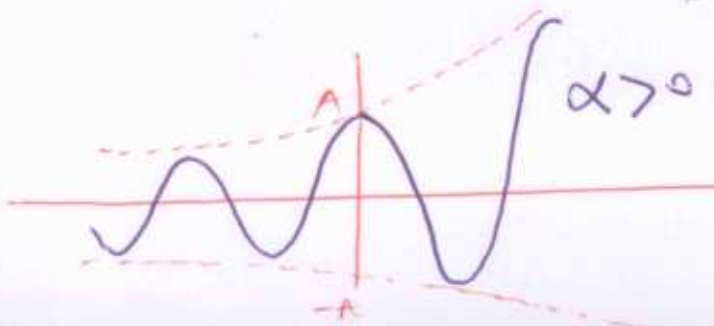
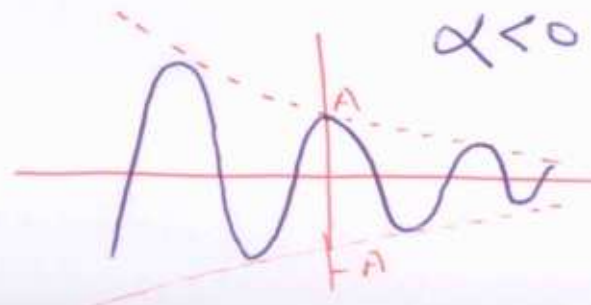
* when ~~***~~ $t = 5t_c \Rightarrow \underline{\underline{x(t) \approx 0}}$

- Complex exponential function -

$$x(t) = A e^{j(\omega t + \alpha)}$$

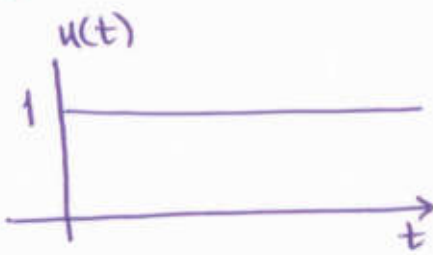
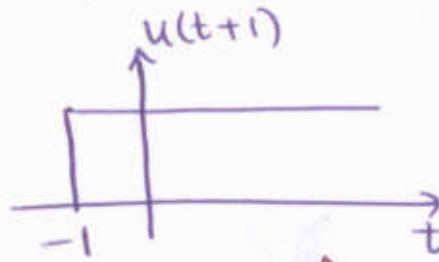
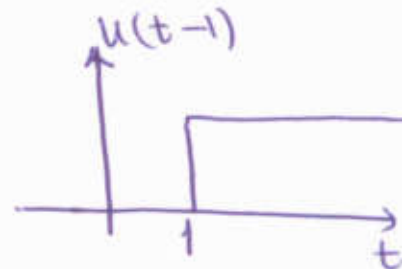
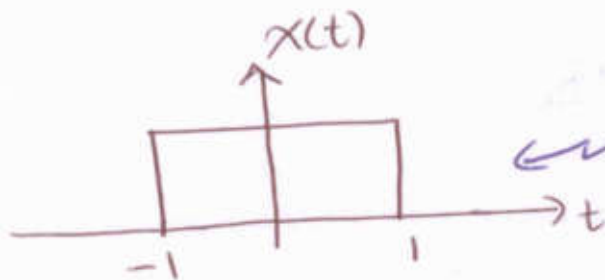
$$x(t) = A e^{\alpha t} \cdot e^{j\omega t} \rightarrow \cos + j \sin$$

A : Amplitude
 α : real part



Examples:

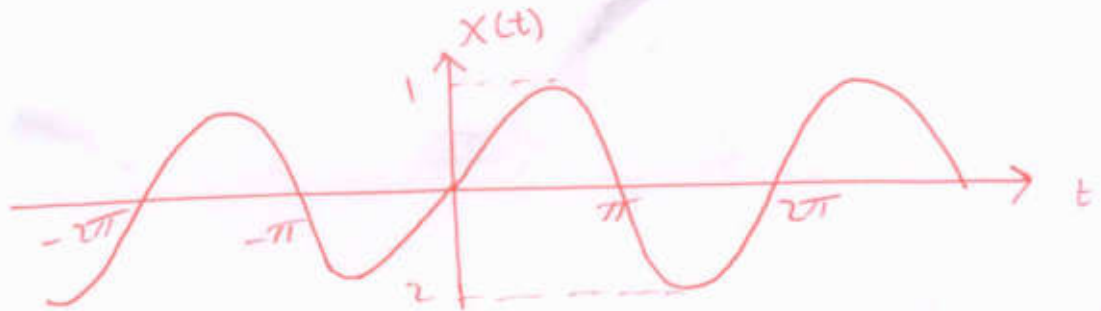
1) $x(t) = u(t+1) - u(t-1)$



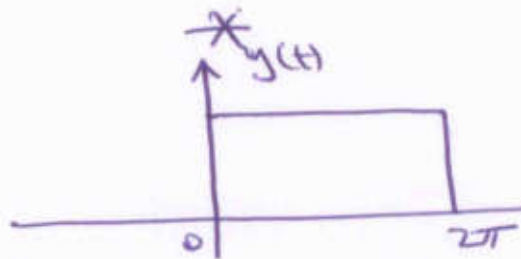
Subtract

is called window func.

2



Find the sin wave function with 2π period.



$$y(t) = u(t) - u(t - 2\pi)$$

$$\therefore x'(t) = \sin \omega t [u(t) - u(t - 2\pi)]$$

• Some properties for Impulse function

(17)

$$1) \int_{-\infty}^{\infty} \delta(t) dt = 1 \quad \& \quad \delta(-t) = \delta(t)$$

$$2) \int_0^{\infty} \delta(t) dt = 1$$

$$3) \int_5^{\infty} \delta(t) dt = 0$$

$$4) \int_5^{\infty} \delta(t-5) dt = 1$$

$$5) \int_{-\infty}^{\infty} x(t) \delta(t-t_0) dt = x(t_0)$$

↳ Constant value.

↑ $x(t)$ (shifting property) $\equiv$ sampling

Example 2

$$1) \int_0^{\infty} \sin(t) \delta(t-3) dt$$

$$= \sin(3)$$

$$2) \int_0^{\infty} \sin(t-2) \delta(t-3) dt$$

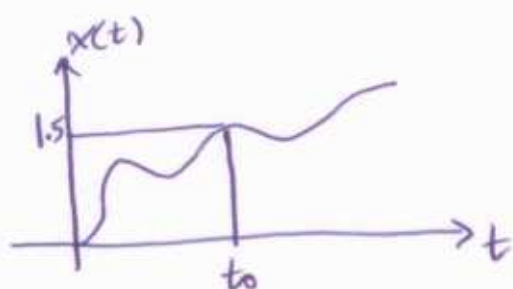
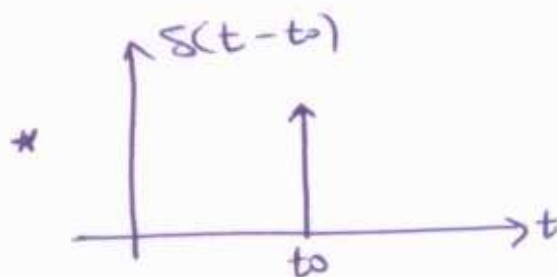
$$= \sin(3-2) = \sin(1)$$

$$3) \int_0^{\infty} \cos(t-\tau) \delta(\tau-2) d\tau \leftarrow \begin{array}{l} \text{هذا هو المتغير} \\ x(\tau) \end{array}$$

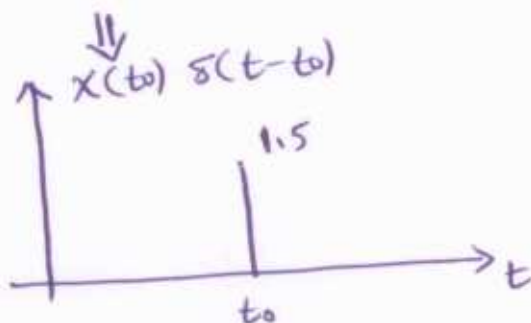
$$= \cos(t-2)$$

#

$$x(t) \cdot \delta(t - t_0) = x(t_0) \delta(t - t_0)$$



discrete is interval Δt is Δt
sampling is Δt



$$x[n] = x[0] \delta[n-0] + x[1] \delta[n-1] + \dots$$

$$x[n] = \sum_{k=0}^{\infty} x[k] \delta[n-k]$$



⊗ Euler's equation 1.

$$e^{j\theta} = \cos \theta + j \sin \theta$$

$$\cos \theta = \frac{1}{2} (e^{j\theta} + e^{-j\theta})$$

$$\sin \theta = \frac{1}{2j} (e^{j\theta} - e^{-j\theta})$$

⊗ Signal representation 2-

- 1) The change in the slope at $t=t_0$ means that ramp function is exist.

$$\boxed{\Delta S r(t-t_0)}$$

slope difference
change point

- 2) The change in Amplitude at $t=t_0$ means that step function is exist.

$$\boxed{\Delta A u(t-t_0)}$$

Amp difference
change point

Ex 2

(1) در هر نقطه تغییر شیب یا دامنه
 (2) تغییر شیب
 (3) تغییر دامنه
 (4) تغییر شیب و دامنه

$$\begin{array}{l} S_1 = 0 \\ S_2 = 0 \\ S_3 = 0 \end{array} \quad \left\{ \begin{array}{l} A_1 = 0 \\ A_2 = 1 \\ A_3 = 0 \end{array} \right.$$

at $t=0$

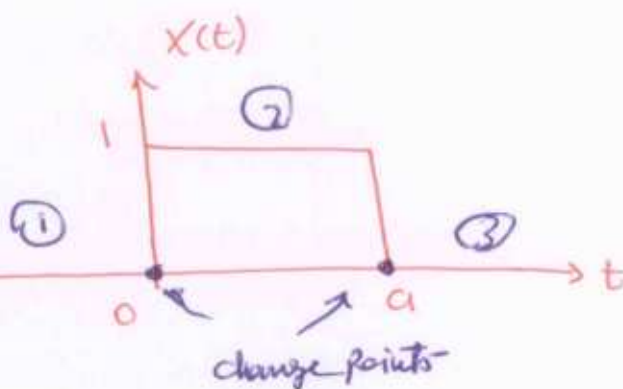
$$(A_{\text{new}} - A_{\text{old}}) u(t - t_0)$$

$$(1 - 0) u(t - 0) \Rightarrow u(t)$$

at $t=a$

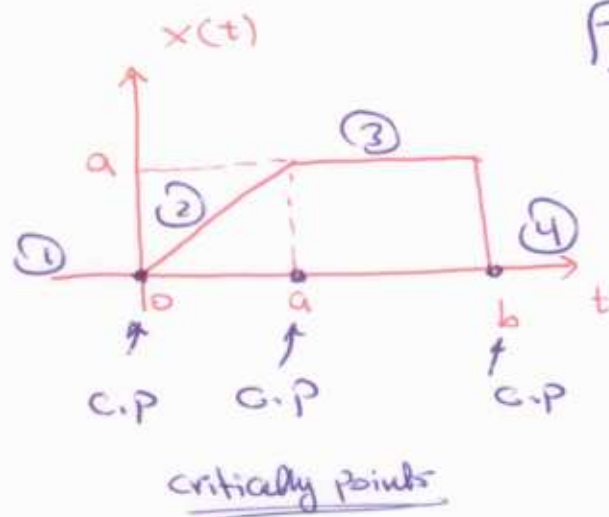
$$(0 - 1) u(t - a) \Rightarrow -u(t - a)$$

$$\therefore x(t) = u(t) - u(t - a) \quad \text{A.}$$



Ex 2

$$\begin{aligned}
 s_1 &= 0 \\
 s_2 &= \frac{a-0}{a-0} = 1 \\
 s_3 &= 0 \\
 s_4 &= 0
 \end{aligned}
 \left\{
 \begin{aligned}
 A_1 &= 0 \\
 A_2 &= 1 \\
 A_3 &= a \\
 A_4 &= 0
 \end{aligned}
 \right.$$



at $t = 0$

$$(1-0)r(t-0) = r(t)$$

at $t = a$

$$(0-1)r(t-a) = -r(t-a)$$

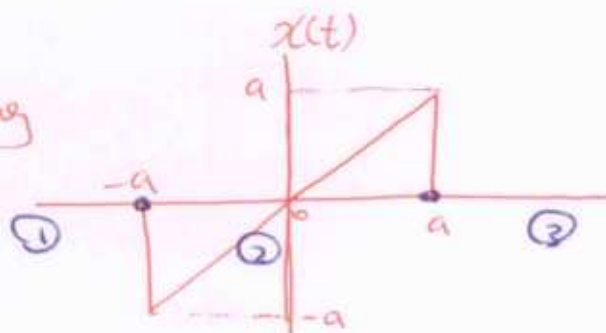
at $t = b$

$$(0-1)u(t-b) = -u(t-b)$$

$$\therefore x(t) = r(t) - r(t-a) - u(t-b) \quad \#$$

Exam

Find the expression of the following signal.



$$\begin{aligned}
 s_1 &= 0 & A_1 &= 0 \\
 s_2 &= \frac{a+a}{a+a} = 1 & A_2 &= -a \\
 s_3 &= 0 & A_3 &= 0
 \end{aligned}$$

at $t = -a$

$$(1-0)r(t+a) + (-a-0)u(t+a) = r(t+a) - au(t+a)$$

at $t = a$

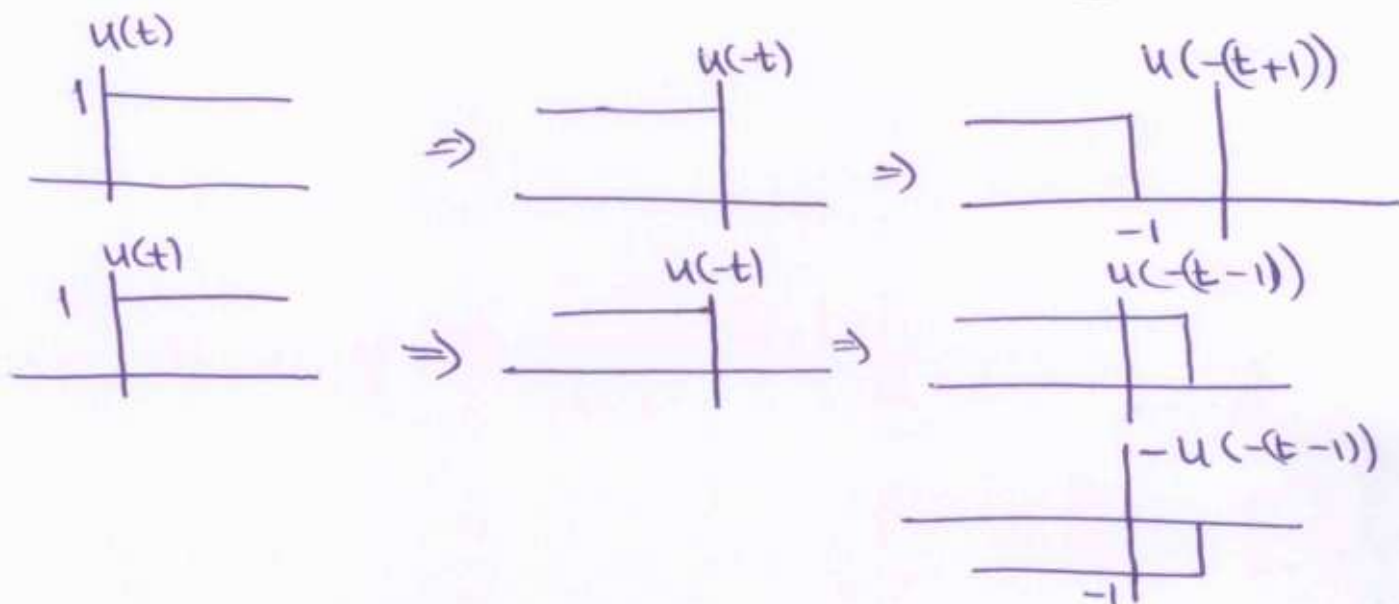
$$(0-1)r(t-a) + (0-a)u(t-a) = -r(t-a) - au(t-a)$$

$$\therefore x(t) = r(t+a) - au(t+a) - r(t-a) - au(t-a) \quad \boxed{21}$$

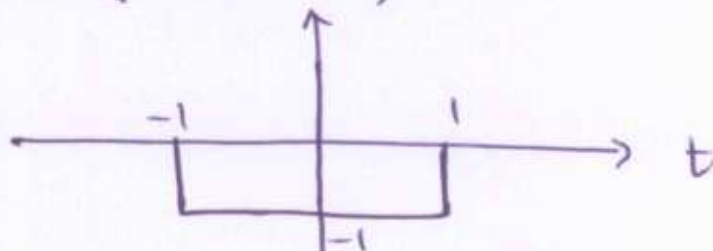
Ex2 Plot the following expression

$$\triangleright x(t) = t [u(-t-1) - u(-t+1)]$$

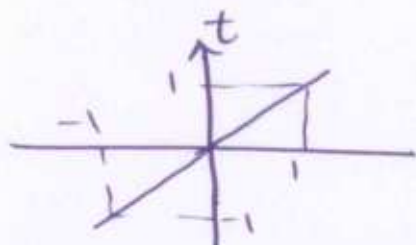
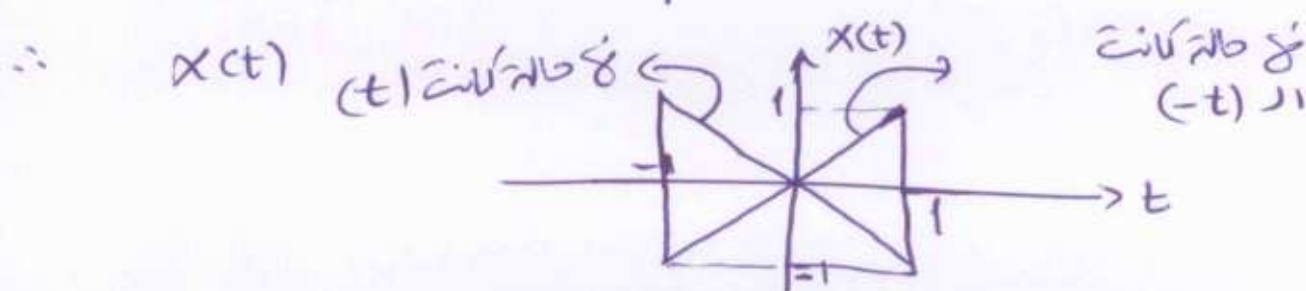
$$x(t) = t [u(-(t+1)) - u(-(t-1))]$$



$$\therefore u(-(t+1)) + (-u(-(t-1)))$$

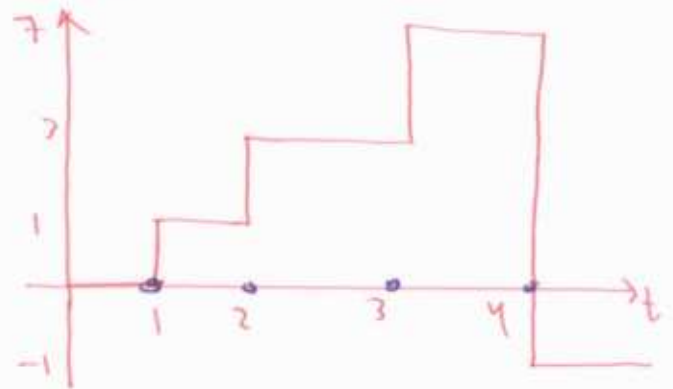


(t)



#

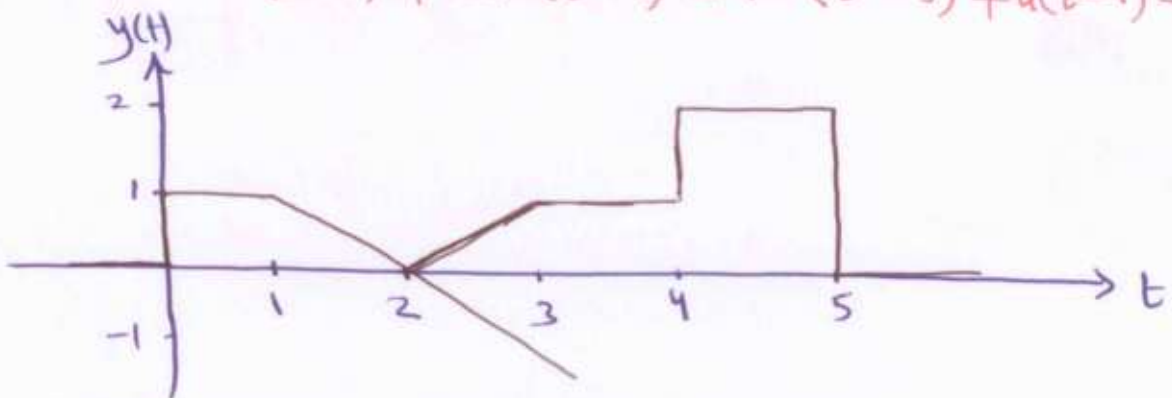
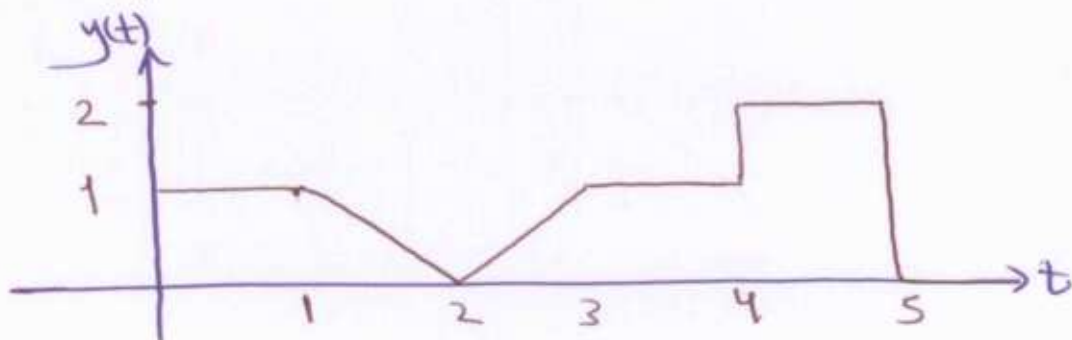
Ex2 Find the expression of $x(t)$?



$$x(t) = u(t-1) + 2u(t-2) + 4u(t-3) - 8u(t-4) \quad \text{JR}$$

Ex2 plot the following signal:

$$y(t) = u(t) - r(t-1) + 2r(t-2) - r(t-3) + u(t-4) - 2u(t-5)$$



JR

notes

in DT signals their some properties:

(23)

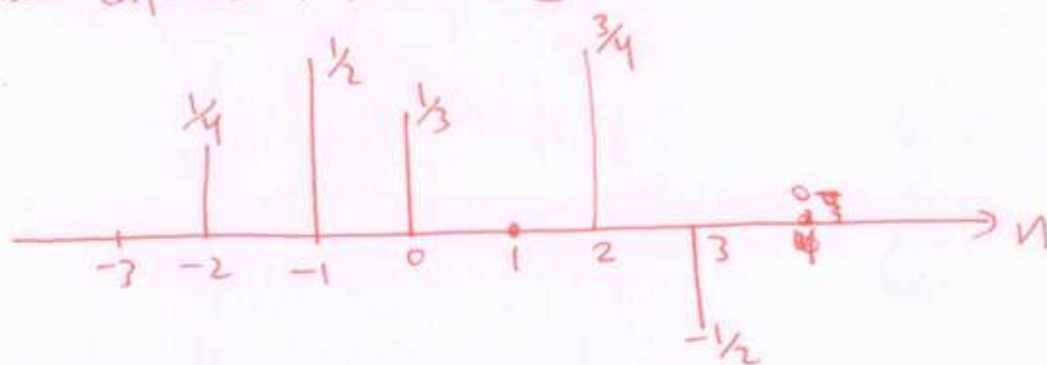
1) $\sum_{n=0}^{\infty} \delta[n-k] = u[n]$ step function

Discr. $\rightarrow$ δ contn. $\rightarrow$ δ LP

2) $x[n] - x[n-1] = \delta[n]$



Ex 2 Find the expression for $x[n]$



$$x[n] = \left\{ \frac{1}{4}, \frac{1}{2}, \frac{1}{3}, 0, \frac{3}{4}, -\frac{1}{2} \right\}$$

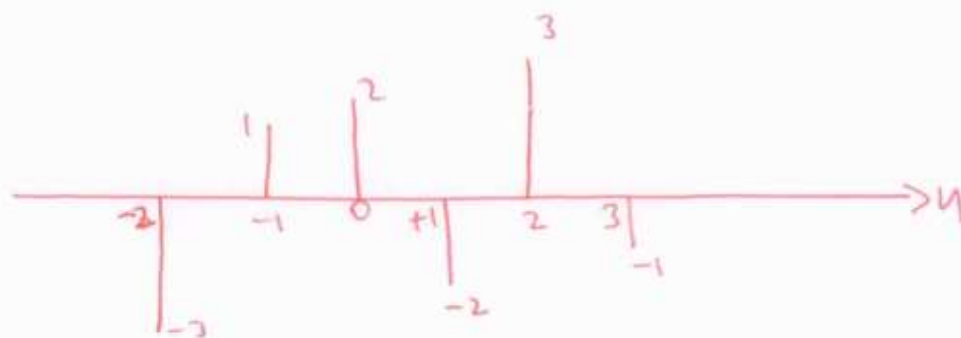
هذه الـ δ ، عدد نكتبه في x -axis $\rightarrow$ δ $\rightarrow$ x -axis $\rightarrow$ δ $\rightarrow$ x -axis $\rightarrow$ δ $\rightarrow$ x -axis

So $x[n] = \frac{1}{4} \delta[n+2] + \frac{1}{2} \delta[n+1] + \frac{1}{3} \delta[n] + \frac{3}{4} \delta[n-2] - \frac{1}{2} \delta[n-3]$

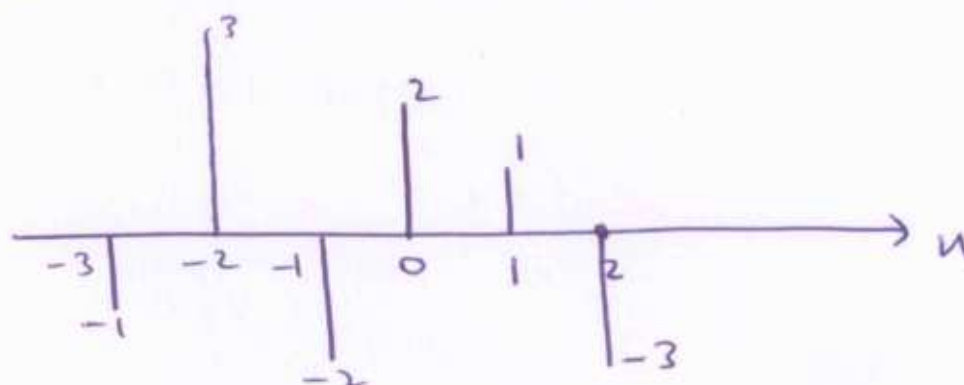
AP

Ex: Find $x[2-n]$, where $x[n]$ shown below

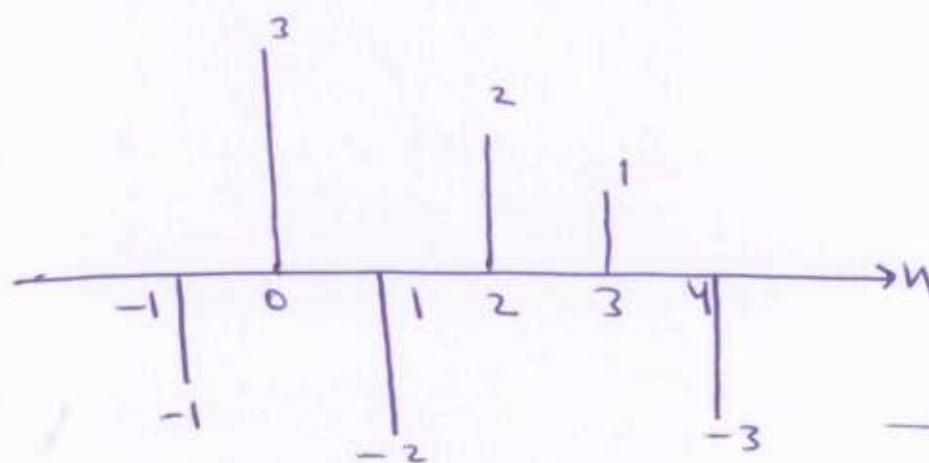
(24)



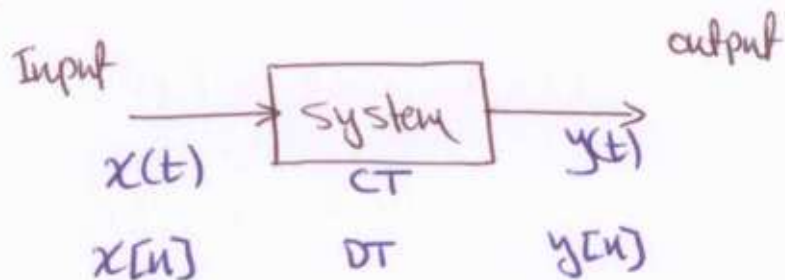
- $x[-n]$ → Reflection about zero point



- $x[-(n-2)]$



~~HP~~

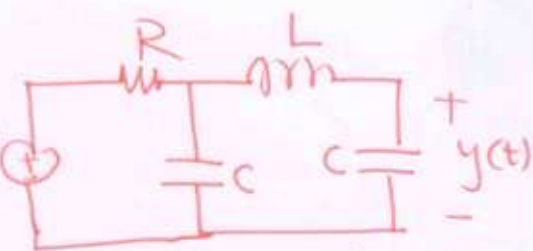


→ the system may be physical components (hardware) or may consist of algorithmic that compute the output signal from input so it called (software)

→ A system is characterized by its input, & its output, and the rule of operations adequate to describe its behavior.

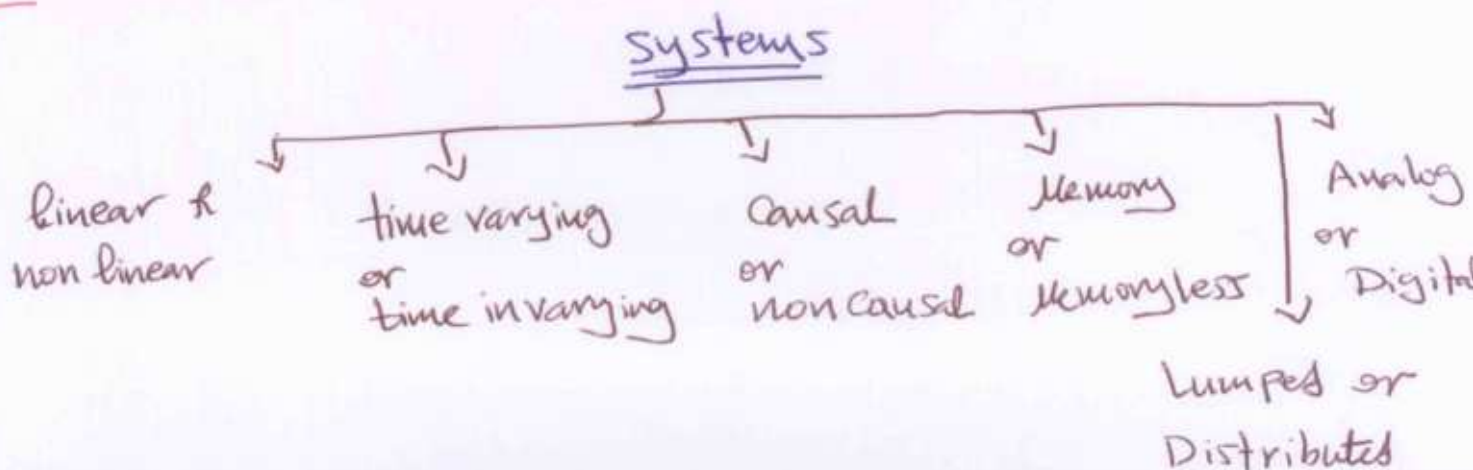
→ Example: RLC circuit

By deriving a mathematical equation that relate the input signal $x(t)$ with output signal $y(t)$, we can find a mathematical model for the system or transfer function



$$H(s) = \frac{Y(s)}{X(s)}$$

1.7 classification of systems:-



① linear and non linear systems

(27)

- To test if the system linear or not that called linearity test.
- There steps for this test which are:
 - 1) $x_1(t) \rightarrow y_1(t) \Rightarrow ax_1 = ay_1$ homogeneity test
 - 2) $x_2(t) \rightarrow y_2(t)$
 - 3) $x_3(t) = ax_1(t) + bx_2(t) \rightarrow y_3(t)$ — ①
 - 4) if $y_3(t) = ay_1(t) + by_2(t)$ — ②

} Additivity test

if ① = ② $\Rightarrow$ then the system is linear

Ex:

at Input $\rightarrow$ output

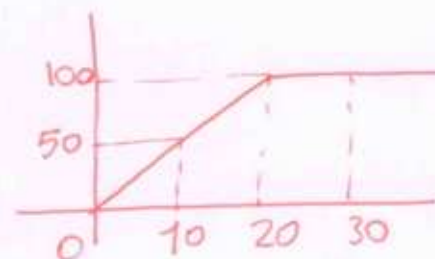
0 $\rightarrow$ 0

10 $\rightarrow$ 50

+
20 $\rightarrow$ 100

30 $\rightarrow$ 150

30 $\rightarrow$ 100 $\neq 150$



so the system non linear

the super position not hold

Ex2

$$y' + 10y + 5 = x$$

(28)

Is this system linear?!

$$\text{let } x_3(t) = ax_1(t) + bx_2(t) \Rightarrow y_3(t) = ay_1(t) + by_2(t)$$

$$ax_1(t) = ay_1' + 10ay_1 + 5a \quad \text{--- ①}$$

$$bx_2(t) = by_2' + 10by_2 + 5b \quad \text{--- ②}$$

$$ax_1 + bx_2 = \cancel{[a+b][y_1' + y_2']} + 10[ay_1 + by_2] + 5[a+b]$$

$$x_3(t) = y_3' + 10y_3 + 5[a+b]$$

المطلوب هو ثابت

∴ the superposition does not hold so the system is not linear

[2] Time Varying & Time invariant :-

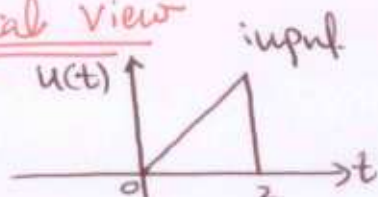
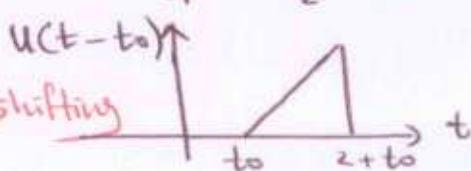
Physical view :-

- if the characteristic of a system do not change with time

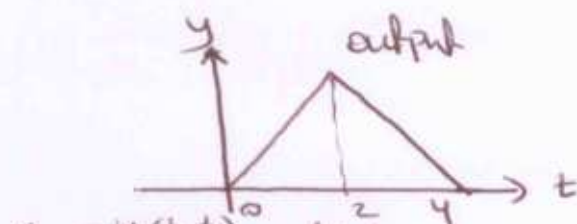
So this system called time Invariant

- The Fan is Time invariant, but the Space rocket is time ~~variant~~.

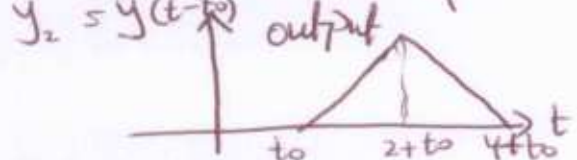
Technical view



$\Rightarrow$



$\Rightarrow$



Invariant input و output لا يتغير مع الزمن

So

if $x(t) \rightarrow y(t)$

$$x(t - t_0) \rightarrow y(t - t_0)$$

So the system called time Invariant

[3] Memory and memoryless :-

- A system is said to be memoryless or instantaneous if the present value of output depends only on the present value of input. [Resistor

like $V(t) = R i(t)$

- A system is said to be memory or dynamic if and only if the present value & future value are affected by past value like [capacitor & inductor]

present value $y(t) = x(t) + x(t-1)$ past value

Ex2

$$y(t) = x(t) \rightarrow \text{memoryless}$$

$$y(t) = x(t-1) \rightarrow \text{memory}$$

$$y(t) = x(t+1) \rightarrow \text{memory}$$

[4] Causal and non causal systems :-

[30]

- A system is called a causal or non causal system if its current output depends on past and current inputs but not on future input. ~~or~~ if it depends on future respectively.
- By another way causality mean the system causal if the output at any time " t_0 " depend only on value of x input signal for $t \leq t_0 \Rightarrow$ mean present time or past

Examples:

$$y(t) = x(t)$$

causal

$$y(t) = x(t-1)$$

causal

$$y(t) = x(t+1)$$

non causal

$$y(t) = x(-t)$$

non causal

الوقت لا يتغير

$$y(t) = x(2t)$$

non causal

الوقت يتغير

non causal

causal

Notes:

every memoryless systems are causal system but not vice versa

Ex: 1) $y(t) = \cos t$ time variant or Invariant 31

$$\cos(t - t_0) = \cos(t) \quad \text{periodic}$$

$$y(t - t_0) = y(t)$$

$\therefore$ system is time invariant

2) $y(t) = t \cos(t)$

$$y_1 = t \cos(t)$$

$$y_2 = t \cos(t_2)$$

$$t_2 = t - t_0$$

$$y_2 = t \cos(t - t_0) \quad \text{--- ①}$$

$$y(t - t_0) = (t - t_0) \cos(t - t_0)$$

$$\therefore y_2 \neq y(t - t_0) \quad \therefore \text{time variant}$$

⊛ Step by step for Determine the time variant or not:

step 1: let $y_1(t)$ output for input $x_1(t)$

step 2: let $x_2(t) = x_1(t - t_0)$ and obtain $y_2(t)$

step 3: from step 1 find $y_1(t - t_0)$

step 4: if $y_2(t) = y_1(t - t_0)$

$\Rightarrow$ or equal if yes the system is Time invariant

if No the system is Time variant

Ex 1 Is the following system's linear, causal, Timevarying memory? 32

1) $y(t) = 10x(t+2) + 5$

- linearity $ax_1 + bx_2 = x_3$
 $ay_1 + by_2 = y_3$

~~*~~
 $ay_1 + by_2 = 10[ax_1(t+2) + bx_2(t+2)] + 5[a+b]$

$$y_3 = 10x_3 + 5[a+b]$$

Superposition not hold $\rightarrow$ non linear system

- non causal because it depend on future $(t+2)$

- Time variant or not

1) $y_1(t) = 10x_1(t+2) + 5$

$$y_2(t) = 10x_1(t-t_0+2) + 5$$

$$y_1(t-t_0) = 10x_1(t-t_0+2) + 5$$

$$\therefore y_2(t) = y_1(t-t_0)$$

So the system is time invariant.

- with memory

$$[2] \quad y(t) = 2x(t) + 3$$

- causal
- memory less
- non linear
- time invariant.

$$[3] \quad y(t) = A t x(t)$$

- Causal
- memory less
- linear
- time varying.

$$[4] \quad y(t) = 2x[n-6] + 6$$

- causal
- non linear
- Time invariant
- memory

In our study we just focus on LTI system
As we will be see in next chapters

END (prepare yourself to quiz)
next lecture